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Conceptual Knowledge of Mathematics Teachers in Equation Concept

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Abstract

This research is backgrounded by a few studies that show that there are mathematics teachers who do not have a good understanding of the content of the mathematics they teach. For this reason, the study aims to analyze the teacher's conceptual knowledge, especially about the concept of equations. The respondents to this study were mathematics teachers at the State Junior High School (SMPN) who taught class VII in a district in Banten province totaling 22 people. The research is processed with descriptive qualitative. The results showed that the teacher's knowledge of equations was obtained 1) doing practice questions but without understanding; 2) obtained by rote memorization; 3) applying in daily activities; 4) interaction with students. In addition, the teacher also has a good understanding of the form of equations.

Introduction

The role of the teachers is very important in education. Professional and good quality of teachers can produce the best future people who can compete in this globalization era. To become the best quality educator, the teacher should have some knowledge on content and pedagogic.

There are two knowledge that should be acquired when someone is learning mathematics; conceptual and procedural knowledge (Attorps, 2006; Khashan, 2014; Rittle-Johnson & Schneider, 2015 ; Zuya, 2017). Conceptual knowledge is explicit and implicit understanding of some principles which arrange the domain and relationship between the concepts in the domain (Rittle-Johnson & Alibali, 1999), while procedural concept is the sequence of acts to solve problems (Kesumawati, 2008; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Schneider, 2015). Those two kinds of knowledge are important components in understanding mathematics (Khashan, 2014), and cannot be separated (Star & Stylianides, 2013). If one of the two knowledges does not exist, so someone cannot understand the concept of Math very well (Rittle-Johnson & Schneider, 2015).

Conceptual and procedural knowledge should be acquired by the Math teachers. The teachers who understand the Conceptual and procedural knowledge very well can give clear explanation and ask questions that can help students to develop their understanding in conceptual which is rich with some contents (Attorps, 2006). According to Shulman (1987) professional teachers are not only acquired procedural knowledge, but also acquired content and are able to explain

the reason why particular things are done so that the teachers can do some reflections on what they have done. Procedural and conceptual knowledge should be understood by the teachers, those knowledges are as follows: (a) knowledge about form and sintax of the system of symbol representation and (b) knowledge about rules and algoritm. These knowledges are very beneficial in completing Mathematics tasks and conceptual knowledge related with the knowledge of mathematics' structure as the basic knowledge (Khashan, 2014; Rittle-Johnson & Alibali, 1999; Zuya, 2017). Therefore, the teaching of mathematics should include the teaching of procedural and conceptual knowledge (Attorps, 2006).

Meanwhile, teaching and achievements in mathematics have been criticized in several countries in the last decades (Attorps, 2006). One of the critics is about the research in mathematics which always focus on developing algorithmic skill rather than in understanding mathematics concept knowledge (Crooks & Alibali, 2014). Several studies have shown that most students deal with mathematical content as procedural knowledge without focusing on conceptual knowledge (Khashan, 2014).

Besides that, a number of studies have shown that most teachers do not have a good understanding of the mathematics content that they teach to high school students (Khashan, 2014). The teachers at high school have reached the average score of conceptual knowledge and cannot use facts and simple relationship. Fewer time and attention toward conceptual knowledge than procedural knowledge (Attorps, 2006; Crooks & Alibali, 2014; Khashan, 2014).

Therefore, in the teaching-learning process the students are less encouraged to develop their thinking skills (Kesumawati, 2008). Especially in the teaching-learning process in the classroom, the students are directed to use formulas and memorize formulas, mathematics is only for doing problems, it is rarely taught to analyze and use mathematics in everyday life (Khashan, 2014; Star & Stylianides, 2013). When the students are taught by doing some different exercise from the one they have learnt, so they will do the same mistakes (Kesumawati, 2008; Star & Stylianides, 2013). Besides that, some findings show that when the teachers ask students to perform a procedure such as solving an equation, students are able to follow the example and get the correct answer without understanding how or why the process works (Attorps, 2006). Attrops also stated that students who can give correct answer will show about how the process of mathematics work but cannot explain the deepest meaning of what the result of the test.

Conceptual knowledge is generated by previous experience and conceptions and by recalling of tasks in which concept definitions have been tested in the process of teaching and learning in educational background. (Tall dan Vinner 2011, Vinner 2010). The experience of mathematics in everyday life and the experience of learning mathematics are an important role in our mathematical thinking. Both provide a limited conception of the nature of mathematics (Hatano, 2016).

Whenever the teachers teach mathematics, they often return to methods learned from school time or their own experiences (Raymond & Santos 2015; Hill 2008). Therefore, a learner's personal experience includes the type of learning experience that has an influence on a person's conceptual knowledge of mathematics.

According to Malinen (2008), knowledges are from someone's life experiences. The experiences can be a conception, false theories, and limited perspectives. There are some misunderstandings and difficulties in learning scientific concept depends on the experience and conception of one's daily life (Duit, 2015).

Mathematical concept being chosen is an equation concept. An equation is a mathematical statement in the form of a symbol that states that two things are the same. The equation concept is chosen because mathematics is the basic knowledge in a wide range of scientific field.
The concepts have been started since early years at junior high schools or SMP. The kinds of equation include 1) Linier; 2) quadrat; 3) absolut; 4) circle; 5) Exponent; 6) algorithm; dan 7) Trigonometry. Based on the background put forward by the researcher, the researcher analysed the mathematics teacher's conceptual knowledge about the concept of equations.

Method

This is descriptive qualitative. The subject of the study is chosen through purposive random sampling. The Math teachers at Junior High School who teaches class 7^{th} at Tangerang Selatan district. The seventh-grade students are chosen from the concept of equation, which is introduced and learned by the students, 2) Respondents are the English department of Math Education. It is considered that the students have understood equation. ;3) the teacher had teaching experience about 10 years. In the practice at the field, one teacher is responsible, one teacher and the other teachers are chosen about 22 years old.

At the early stage of the study, the researcher collaborated with Dinas Pendidikan Tangerang Selatan districts, to get some knowledge. Because the research was conducted during the Covid-19 Pandemic, where the learning process changed from face-to-face to distance learning, data collection was carried out online. Data collection was carried out by giving tests and assignments, interviews.

Results and Discussion

The respondent in this study is the mathematics teacher at Junior High School who teaches class VII at Tangerang Selatan district. One teacher from every junior high school at Tangerang Selatan is chosen as respondents of the study. The sample of the respondents are 22 teachers. At the first stage, the teachers who have returned the online test are 16 or about 73%. Here is the description of the respondents.

Gender

Tabel 1. Gender

Gender	**%**
Female	54,7
Male	45,3
Total	100,0

The result of analysis showed that female respondents are more than male respondents, that is 54.7 %. The differences are about 9.4%.

Age

Table 2. Age of Respondent

Age	%
< 30 Years	5,7
31 - 35 Years	37,7
36 - 40 Years	43,4
> 40 Years	13,2
Total	100,0

The result of analysis showed that the respondent who are between 36-40 years old are the highest 43.4%. By the age between 31-35 years old are 37.7% respondents and then, by the age of under 30 years old are 5.7% and respondent who are more than 40 years old are 13,2% respondents.

Education

Tabel 3. Education

Education	%
Diploma/S1/undergraduate	71,7
S2/graduate	28,3
Total	100,0

The result of the analysis showed that educational background of respondents are diploma (71.7%) and 28.3% are graduate or S2 (28.3%).

Years of Service

Tabel 4. Years of Service

Years of Service	%
1 - 5 years old	24.5
6 - 10 years old	20.8
10 - 15 years old	41.5
> 15 tahun	13.2
Total	100.0

The result of analysis showed that the respondents have worked around 10-15 years and it was 41.5%, and the respondents who have worked for 1-5 years are 24.5%. while 20.8% respondents have worked for 6-10 years, and then 13.2% respondents have worked for more than 15 years.

Amounts of Students

Tabel 5. Amounts of Students

Amounts of Students	%
1 - 15 students	0
16- 30 students	100,0
> 30 students	0
Total	100,0

In this section, the researcher discusses about how the teachers understand about equation. In this case the researcher analyzes the results of interviews and tests on the theme: experience, understanding of the form of equations, and definition of concepts. Here is the explanation.

Equation Learning Experience

The first question is related to the experience of studying mathematics at university. For the theme of experiences about learning mathematics, researchers categorize in three categories, namely

Category 1: Teaching-learning process as a tool for an interaction with other students
The teacher's conception showed that they have mathematical knowledge about equations obtained through interaction with students. In this case, mathematical knowledge is acquired in a communicative context.

> *"At university we have small groups where we discuss and solve some problems. That's when I started to understand how to use equations" (C)*

> *"When I was in college, I discussed and contemplated the problems in mathematics with my friends. I can ask other students and discuss with them" (B)*

Category 2: Learning Equation by doing regular practices
The first category showed that the students' understanding on equation can be achieved by doing some regular exercises. In this case, the students do the exercises without deep understanding. Here are the examples of respondents.

> *"I learned by connecting the concept of equations to other subjects. Focus on the learning object itself. (C)*

> *... I must understand first with the new concept i can answer the question... next I apply it in everyday life" (H)*

> *... I understand first about the concept of equations, then I can connect with other sciences. For example, I link mathematics in physics and physics in mathematics... (I)*

Category 3: Learn equations by doing regular questions
The third category shows that the teacher's knowledge of equations is obtained by doing routine questions that are usually given. In this case, learning is understood as an exercise in working on questions but without understanding. Here is an example of the respondent's expression.

"I continue to practice doing math problems without understanding the material" (D)

"... i learned math through my mother's practice questions" (C) "When I was in college, I studied mathematics including equations by practicing the problems given continuously" (A)

Category 4: Learning equations by memorization

The fourth category indicates that mathematical knowledge is obtained by means of memorization. Here are the respondents' phrases.

"... I'm busy, don't have time to study... So I memorized the sample questions and answers in order to pass the exam. (F)

"I don't remember if I understood what I was working on because I was used to memorizing formulas. I memorize it sometimes I forget because I don't understand. (L)
"... sometimes I don't understand, so I memorize the whole formula and the steps of working on the problem" (M)

Understanding

To find out students' knowledge about understanding the form of equations, respondents were given the following questions. The question is whether the question in the "question" table is an equation or not. Here is a table of respondents' questions and answer percentages.

No	Test Items	Correct Answers
1	$x^2 - 5x - 10$	95 %
2	$2x + 5y = \sqrt{a}$	88 %
3	$(x - \sqrt{2})(x^2 - 9)(5 + x)$	83 %
4	$a + 5b = 6a + 9{,}5b$	83 %
5	$(x^2) + (y - 1)^2 = 25$	72 %
6	$a^b = \sqrt{a\sqrt{a\sqrt{a}}}$	70 %
7	$X = 2$	67 %
8	$e^{x+y} = 1$	67 %

9	$\int f(x)dx = x^2 + C$	44 %
10	$f(x) = 2x + 1$	57 %
11	$cos^2 \propto + sin^2 \propto = 1$	51 %
12	$y''(x) + y(x) = 0{,}5cos2x$	55 %
13	$a^{lne} = e$	55 %
15	$x + \|x - 3\| \geq \|x - 1\| + 2$	55 %

Source: Referring to Attrop (2006)

From the respondents' answers the correct one is for as much as 95%. Meanwhile, the percentage of edging is 55%. Here are the results of the interview after the respondent answered the question, which has been divided into several categories.

Category 1: Equations as Procedures
In the first category, the teacher's conception of equations has focus on the process of solving the equations themselves and mathematical symbols.

> *"... I'm not sure if this can be considered an equation... I am confused as to whether that a=3 and b=2d natural statements a+5b=6a+9.5b". (E)*

> *".... I feel that there are three unknown variables on the question of 2x + 5y = a. I can't assume it's an equation because I don't know the indigo"(F).*

In the first category, respondents focused on the process of solving the equation itself and on mathematical symbols rather than the mathematical content itself. Some teachers have difficulty in recognizing statements because equations do not look like 'ordinary equations'.

Category 2: Equations as answers
The conception in the second category suggests that the teacher's attention is focused not only on the settlement procedure but also the arithmetic interpretation of the equal sign.

> *"I understand this as an answer.... The value of an unknown factor is already given. That's why it's not an equation". (H; x = 2)*

> *"No, it's just an expression for x". (L; x = 2)*

> *"Bis the equation... it's just an answer" (K; ex + y = 1)*

> *"No, this is a rule or a formula. " (A; cos 2α + sin 2α = 1)*

The conception in the second category suggests that arithmetic understanding of the equal sign is the focus of the teacher's attention. In the conception of such statements is not understood as an equation, since the teacher thinks that the equation is already solved. The conception of equality reveals that the teacher has a process-oriented conception. It seems that teachers understand the algebra of ideas as a process rather than an abstract object.

Conceptual Definition

The third sub-question of the second research question is:
How do teachers describe similarities?
The teachers explain the equations as concrete illustrations; as a tool for knowing the unknown; as an equation between two quantities and as a transition to algebraic thought.

Category 1: Equations as a tool for finding out the unknown
The teacher's attention to this category is focused on the action of the process of solving equations rather than the concrete metaphor of equations, which is so dominant in the first category. For example, the following quote describes the conception of the teacher in the second category:

> *"Equations are a way to find these unknown values. (C)*
>
> *"Equations are tools for solving problems. (D)*
>
> *" Equations are solving unknown numbers. (A)*

Category 2: Equations as equations between two quantities
Here the teachers emphasize not only on the process of solving equations but also the structure of equations. The following excerpts describe key aspects of the third category:

> *The equation is that both sides are equal. (I)*
>
> *It means the equation between the left side and the right side... and must solve an unknown number. (J)*
>
> *The left side is equal to the right side and then you must find an unknown amount. I have never reflected on what the concept of equation means... 7 + x = 9, roughly like this.... (E)*

Conclusion

The results of the analysis showed that the teachers understood the equations by doing routine questions, memorizing and applying them to other sciences. As for the form of equations, the teacher's conception has a focus on the process of solving the equation itself and mathematical symbols. In addition, the teacher's attention is focused not only on the settlement procedure but also the arithmetic interpretation of the equal sign.

The teachers explain the equations as concrete illustrations; as a tool for knowing the unknown; as an equation between two quantities and as a transition to algebraic thought. In addition, the teachers also stated equation as a tool for finding out the unknown and p ersamaan as an equation between two magnitudes.

Recommendations

The recommendation of this study is the need for a Professional Deevelopment Program design which is expected to build conceptual knowledge and procedural ability of mathematics teachers about equation concepts. The research design used is Design Didactical Research (DDR) which is a design to help a person in conceptualizing and facilitating the learning process.

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